# **3-D Electromagnetic Field Analysis Combined with Mechanical Stress** Analysis for Interior Permanent Magnet Synchronous Motors

Katsumi Yamazaki, IEEE Member, and Akihiro Aoki

Chiba Institute of Technology, Chiba, 275-0016, Japan, yamazaki.katsumi@it-chiba.ac.jp

The method of 3-D electromagnetic field analysis combined with mechanical stress analysis has been developed for loss calculation of interior permanent magnet motors. Both the stator compressive stress caused by shrink fitting and the rotor tensile stress caused by centrifugal force are considered in the stress analysis. The 3-D flux density vector is calculated by the electromagnetic field analysis including the core end regions. The variation in the stress effect with the angle between stress and flux vectors is taken into account by using equivalent stress. The calculated loss is compared with the experimental loss in order to confirm the validity of the analysis. It is clarified that not only the compressive stress caused by the stator shrink fitting but also the tensile stress caused by the rotor centrifugal force affect the electromagnetic field in the interior permanent magnet motors.

Index Terms-Permanent magnet motors, stress, losses, finite element methods.

#### I. INTRODUCTION

T is well known that the mechanical stress causes characteristics deterioration of electrical steel sheets [1]-[5]. The permeability decreases, whereas the core loss increases with the stress. In the application for rotating machines, most famous effect is the increase in reluctivity and core loss of stator yoke with compressive stress caused by shrink fitting. Many papers have dealt with this phenomenon. In this case, the direction of the flux density vector at the yoke is nearly parallel to that of the principal stress axis. As a consequence, the variation in the stress effect with the angle between the stress and flux density vectors is neglected in most of papers.

On the other hand, large mechanical stress is also generated at the rotors of interior permanent magnet synchronous motors (IPMSMs) at high speeds because of the centrifugal force due to rotation [4]. As this stress is in tensile direction, it has been considered that the characteristics deterioration of the rotor core by this stress is relatively small. However, in this case, the flux density vector is not parallel to the principal stress axis. As a consequence, it is considered that the magnetic characteristics of the rotor core are deteriorated by transvers stress effect. In reference [5], we investigated this effect by using 2-D analysis. However, 3-D analysis is required for accurate estimation at core-ends.

From these viewpoints, we have developed the method of 3-D electromagnetic field analysis combined with mechanical stress analysis. Both the stator compressive stress caused by shrink fitting and the rotor tensile stress caused by centrifugal force are considered in the stress analysis. The 3-D flux density vector is calculated by the electromagnetic field analysis including the core end regions. The variation in the stress effect with the angle between stress and flux vectors is taken into account by using equivalent stress [6]. The proposed method is applied to a high speed IPMSM in order to confirm the validity.

### II. CALCULATION METHOD

The mechanical stress caused by the stator shrink fitting and the rotor centrifugal force can be approximated by 2-D equations, as follows:

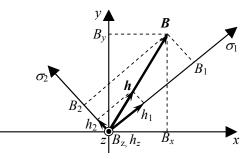


Fig. 1. Principal stress axis and flux density vector.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x = 0 \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0$$
<sup>(2)</sup>

where  $\sigma_x$  and  $\sigma_y$  are the *x* and *y* components of the stress, respectively;  $\tau_{xy}$  is the shearing stress;  $f_x$  and  $f_y$  are the x and y components of centrifugal force. The 2-D plane stress finite elment analysis (FEA) is applied to slove these equations.

On the other hand, the 3-D edge FEA is applied to the electromagnetic field analysis in order to take the axial z component of flux density at the core ends. Fig. 2 shows an example of flux density vector in motor cores and principal stresses ( $\sigma_1$ ,  $\sigma_2$ ) calculated from  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$ . In the figure, the vector magnetic properties are neglected. In general, ( $B_x$ ,  $B_y$ ,  $B_z$ ) and ( $\sigma_1$ ,  $\sigma_2$ , 0) are not parallel even in the case of isotropic magnetic materials. Therefore, 2-D Maxwell-Ampere equation at the motor cores can be expressed, as follows:

$$\nabla \times \left\{ \nu \left( \sigma_1, \sigma_2, B_x, B_y, B_z \right) \nabla \times A \right\} = 0$$
(3)

where *A* is the magnetic vector potential;  $\nu$  is the reluctivity, which is a function of  $\sigma_1$ ,  $\sigma_2$ ,  $B_x$ ,  $B_y$ , and  $B_z$ . Since enormous experiments are required to obtain this function,  $\nu$  is simplified by using equivalent stress  $\sigma_{eq}$  [6], as follows:

$$\nu(\sigma_1, \sigma_2, B_x, B_y, B_z) = C_{\nu}(\sigma_{eq}, |\boldsymbol{B}|) \nu_{\sigma_0}(|\boldsymbol{B}|)$$
(4)

where  $v_{\sigma 0}$  is the reluctivity when the mechanical stress is zero;  $C_v$  is the reluctivity increase ratio.  $\sigma_{eq}$  is derived under the assumption that a same magneto-elastic energy leads to a same characteristics of the magnetic materials. The energy for ( $\sigma_1$ ,

 $\sigma_2$ ) and  $(B_x, B_y, B_z)$  is equal to that for single axial stress  $\sigma_{eq}$ , which direction is parallel to B [6]. Therefore, v can be expressed by using  $C_{\nu}$ , which is determined by experiments of core materials by imposing single axial stress  $\sigma$  along the direction of **B**. The general expression of  $\sigma_{eq}$  is as follows:

$$\sigma_{eq} = \frac{3}{2} \boldsymbol{h}^{\mathrm{T}} \boldsymbol{s} \boldsymbol{h}$$
 (5)

where h is the unit vector, which direction corresponds to that of the magnetic field and s is the deviatoric part of the stress tensor. In the case of the 2-D plane stress, s is expressed, as follows:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_x & \sigma_y & 0 \end{bmatrix} \begin{bmatrix} \sigma_x + \sigma_y & 0 & 0 \\ 0 & \sigma_y + \sigma_y & 0 \end{bmatrix}$$
(6)

$$\begin{bmatrix} r_{xy} & \sigma_y & \sigma_y \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \sigma_x + \sigma_y & \sigma_z \\ 0 & 0 & \sigma_x + \sigma_y \end{bmatrix}$$

By using the components along the principal axes, where  $\tau_{xy}$ disappears, the equation (5) can be expressed, as follows:

$$\sigma_{eq} = \left(\sigma_1 - \frac{1}{2}\sigma_2\right)h_1^2 + \left(-\frac{1}{2}\sigma_1 + \sigma_2\right)h_2^2 + \left(-\frac{1}{2}\sigma_1 - \frac{1}{2}\sigma_2\right)h_2^2$$
(7)

In this expression, negative values of  $\sigma_1$ ,  $\sigma_2$ , mean the compressive stress, whereas positive values mean tensile stress. Therefore, this expression implies that tensile stress has half the effects of compressive stress on the deterioration of core material characteristics when the magnetic field is in right angle to the stress direction.

The core loss is also calculated by using  $\sigma_{eq}$  due to the method described in [5].

## III. RESULTS AND DISCUSSION

The proposed method is applied to a 100 kW class interior permanent magnet motor. Fig. 2 shows the principal stress distribution obtained by the stress analysis. vector Circumferential compressive stress caused by shrink fitting is observed at the stator voke, whereas large tensile stress caused by centrifugal force is observed at the rotor surface.

Fig. 3 shows the flux density distribution. It is observed that the flux density vector at the rotor surface is almost in radial direction. In addition, the axial component increases at the rotor core end. It is considered that the magnetic characteristics are deteriorated by the tensile stress at these parts. Fig. 4 shows experimental and calculated iron loss. The accuracy is improved by the proposed 3-D method.

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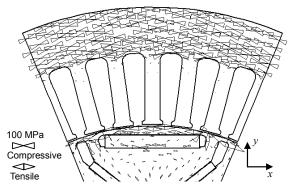


Fig. 2. Principal stress vector distribution (10000rpm).

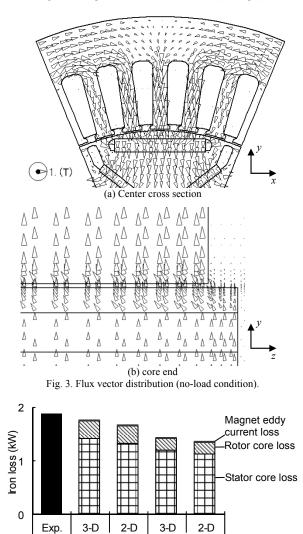


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Without stress

With stress

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